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Zero Mode and Symmetry Breaking on the Light Front [1]

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Abstract

We discuss the spontaneous symmetry breaking (SSB) on the light front (LF) in view of the zero mode. We first demonstrate impossibility to remove the zero mode in the continuum LF theory by two examples: The Lorentz invariance forbids even a free theory on the LF and the trivial LF vacuum is lost in the SSB phase, both due to the zero mode as the accumulating point causing uncontrollable infrared singularity. We then adopt the Discretized Light-Cone Quantization (DLCQ) which was first introduced by Maskawa and Yamawaki to establish the trivial LF vacuum and was re-discovered by Pauli and Brodsky in a different context. It is shown in DLCQ that the SSB phase can be realized on the trivial LF vacuum only when an explicit symmetry-breaking mass of the Nambu-Goldstone (NG) boson m_π is introduced as an infrared regulator. The NG-boson zero mode integrated over the LF must exhibit singular behavior $\sim 1/m_\pi^2$ in the symmetric limit $m_\pi \to 0$ in such a way that the LF charge is not conserved even in the symmetric limit; $\dot{Q} \neq 0$. There exists no NG theorem on the LF. Instead, this singular behavior establishes existence of the massless NG boson coupled to the current whose charge satisfies $Q|0\rangle = 0$ and $\dot{Q} \neq 0$, in much the same as the NG theorem in the equal-time quantization which ensures existence of the massless NG boson coupled to the current whose charge satisfies $Q|0\rangle \neq 0$ and $\dot{Q} = 0$. We demonstrate such a peculiarity in a concrete model, the linear sigma model, where the role of zero-mode constraint is clarified.

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1 INTRODUCTION

Much attention has recently been paid to the light-front (LF) quantization [1] as a promising approach to solve the nonperturbative dynamics. The most important aspect of the LF quantization is that the physical LF vacuum is simple, or even trivial [2]. However, such a trivial vacuum, which is vital to the whole LF approach, can be realized only if we can remove the so-called zero mode out of the physical Fock space ("zero mode problem" [3]).

Actually, the Discretized Light-Cone Quantization (DLCQ) was first introduced by Maskawa and Yamawaki [3] in 1976 to resolve the zero mode problem and was re-discovered by Pauli and Brodsky [4] in 1985 in a different context. The zero mode in DLCQ is clearly isolated from other modes and hence can be treated in a well-defined manner without ambiguity, in sharp contrast to the continuum theory where the zero mode is the accumulating point and hard to be controlled in isolation [5]. In DLCQ, Maskawa and Yamawaki [3] in fact discovered a constraint equation for the zero mode ("*zero-mode constraint*") through which the zero mode becomes dependent on other modes and then they observed that the zero mode can be removed from the physical Fock space by solving the zero-mode constraint, thus *establishing the trivial LF vacuum in DLCQ*.

Such a trivial vacuum, on the other hand, might confront the usual picture of complicated nonperturbative vacuum structure in the equal-time quantization corresponding to confinement, spontaneous symmetry breaking (SSB), etc.. Since the vacuum is proved trivial in DLCQ [3], the only possibility to realize such phenomena would be through the complicated structure of the operator and only such an operator would be the zero mode. In fact the zero-mode constraint itself implies that the zero mode carries essential information on the complicated dynamics. One might thus expect that explicit solution of the zero-mode constraint in DLCQ would give rise to the SSB while preserving the trivial LF vacuum. Actually, several authors have recently argued in (1+1) dimensional models that the solution to the zero-mode constraint might induce spontaneous breaking of the discrete symmetries [6]. However, the most outstanding feature of the SSB is the existence of the Nambu-Goldstone

(NG) boson for the continuous symmetry breaking in (3+1) dimensions.

In this talk I shall explain, within the canonical DLCQ [3], how the NG phenomenon is realized through the zero modes in (3+1) dimensions while keeping the vacuum trivial. The talk is based on recent works done in collaboration with Yoonbai Kim and Sho Tsujimaru [7, 8]. We encounter a striking feature of the zero mode of the NG boson: Naive use of the zero-mode constraints does not lead to the NG phase at all ((false) "no-go theorem") in contrast to the current expectation mentioned above. It is inevitable to introduce an infrared regularization by explicit-breaking mass of the NG boson m_π. The NG phase can only be realized via peculiar behavior of the zero mode of the NG-boson fields: *The NG-boson zero mode, when integrated over the LF, must have a singular behavior* $\sim 1/m_\pi^2$ in the symmetric limit $m_\pi^2 \to 0$. This we demonstrate both in a general framework of the LSZ reduction formula and in a concrete field theoretical model, the linear σ model. The NG phase is in fact realized in such a way that the *vacuum is trivial* while *the LF charge is not conserved* in the symmetric limit $m_\pi^2 \to 0$.

2 ZERO MODE PROBLEM IN THE CONTINUUM THEORY

Before discussing the zero mode in DLCQ, we here comment that *it is impossible to remove the zero mode in the continuum theory* in a manner consistent with the trivial vacuum, *as far as we use the canonical commutator* [3] :

$$[\phi(x), \phi(y)]_{x^+=y^+} = -\frac{i}{4}\epsilon(x^- - y^-)\delta^{(2)}(x^\perp - y^\perp), \tag{1}$$

where the sign function $\epsilon(x^-)$ is defined by

$$\epsilon(x^-) = \frac{i}{\pi}\mathcal{P}\int_{-\infty}^{+\infty} \frac{dp^+}{p^+} e^{-ip^+x^-} \tag{2}$$

[3] We choose the LF "time" as $x^+ = x_- \equiv \frac{1}{\sqrt{2}}(x^0 + x^3)$ and the longitudinal and the transverse coordinates are denoted by $\vec{x} \equiv (x^-, x^\perp)$, with $x^- \equiv \frac{1}{\sqrt{2}}(x^0 - x^3)$ and $x^\perp \equiv (x^1, x^2)$, respectively.

through the principal value prescription and hence has no $p^+ \equiv 0$ mode. The point is that the real problem with the zero mode in the continuum theory is *not a single mode* with $p^+ \equiv 0$, which is just measure zero, but actually the *accumulating point* $p^+ \to 0$ [5].

To demonstrate this, let us first illustrate a no-go theorem found by Nakanishi and Yamawaki [5]. The LF canonical commutator (1) gives explicit expression of Wightman two-point function on LF:

$$\langle 0|\phi(x)\phi(0)|0\rangle|_{x^+=0} = \frac{1}{2\pi}\int_0^\infty \frac{dp^+}{2p^+} e^{-ip^+x^-} \cdot \delta^{(2)}(x^\perp), \tag{3}$$

which is logarithmically divergent at $p^+ = 0$ and local in $x^\perp$ and, more importantly, is independent of the interaction and the mass. It is easy to see that this is wrong, for example, in the free theory where the Lorentz-invariant two-point Wightman function is given at any point x by

$$\Delta^{(+)}(x) = \frac{1}{(2\pi)^3}\int_0^\infty \frac{dp^+}{2p^+} e^{-ip^-x^+ - ip^+x^- + ip^\perp x^\perp} = \frac{m}{4\pi^2\sqrt{-x^2}} K_1(m\sqrt{-x^2}), \tag{4}$$

where K_1 is the Hankel function. Restricting (4) to the LF, $x^+ = 0$, yields $\frac{m}{4\pi^2\sqrt{x_\perp^2}} K_1(m\sqrt{x_\perp^2})$, which is finite (positive definite), nonlocal in $x^\perp$ and dependent on mass, in obvious contradiction to the above result (3). Actually, the Lorentz-invariant result (4) is a consequence of the *mass-dependent* regularization of $1/p^+$ singularity by the infinitely oscillating (mass-dependent) phase factor $e^{-ip^-x^+} = e^{-i(m^2+p_\perp^2)/2p^+ \cdot x^+}$ *before taking the LF restriction* $x^+ = 0$. Namely, there exists *no free theory on the LF*! This difficulty also applies to the interacting theory satisfying the Wightman axioms [5]. Thus the LF restriction from the beginning loses all the information of dynamics carried by the *zero mode as the accumulating point.*

Next we show another difficulty of the continuum LF theory, that is, *as far as the sign function is used*, the *LF charge does not annihilate the vacuum* in the SSB phase and hence the trivial LF vacuum is never realized [8]. Let me illustrate this in the $O(2)$ σ model where the fields σ, π obey the canonical commutator (1), which yields

$$[Q, \sigma'(x)] = -i\pi(x), \quad [Q, \pi(x)] = i\sigma'(x) + \frac{i}{2}v, \tag{5}$$

where the LF charge is defined as usual by $Q = \int d^3\vec{x}(\pi\partial_-\sigma' - \sigma'\partial_-\pi - v\partial_-\pi)$, and the anti-periodic boundary condition (B.C.) is imposed on π and the shifted field $\sigma' \equiv \sigma - v$ ($v = \langle\sigma\rangle$) in order to eliminate the surface terms in (5), $\phi(x^- = \infty) + \phi(x^- = -\infty) = 0$. The non-zero constant term on the R.H.S. of (5) has its origin in the commutation relation (1), which is consistent with the anti-periodic B.C., $\pi(x^- = \infty) = -\pi(x^- = -\infty) \neq 0$ [4]. Then we have

$$\langle 0|[Q, \pi(x)]|0\rangle = i\langle 0|\sigma'(x)|0\rangle + \frac{i}{2}v = \frac{i}{2}v \neq 0. \tag{6}$$

Namely, the *LF charge does not annihilate the vacuum*, $Q|0\rangle \neq 0$, due to the zero mode as the accumulating point, even though we have "removed" exact zero mode $p^+ \equiv 0$ by shifting the field σ to σ'. This implies that there in fact *exists a zero-mode state* $|\alpha\rangle \equiv Q|0\rangle$ with zero eigenvalue of P^+ such that $P^+|\alpha\rangle = [P^+, Q]|0\rangle = 0$ (due to P^+ conservation). Our result disagrees with Wilson et al. [10] who claim to have eliminated the zero mode (to be compensated by "unusual counter terms") in the continuum LF theory.

3 NAMBU-GOLDSTONE BOSON ON THE LIGHT FRONT

In contrast to the continuum LF theory mentioned above, we already mentioned in Introduction that the trivial vacuum is always realized in DLCQ [3]. We now discuss how such a trivial vacuum can be reconciled with the SSB phenomena. Here we use DLCQ [3, 4], $x^- \in [-L, L]$, with a periodic boundary condition in the x^- direction, and then take the continuum limit $L \to \infty$ in the end of whole calculation. We should mention here that the above no-go theorem [5] cannot be solved by simply taking the continuum limit of the DLCQ nor by any other existing method and would be a future problem to be solved in a more profound way.

[4] Without specifying B.C., we would not be able to formulate consistently the LF quantization [9, 8]. If we used anti-periodic B.C. for the *full field* σ, we would have no zero mode (no v) and hence no symmetry breaking anyway. On the other hand, vanishing fields at $x^- = \pm\infty$ contradict the commutation relation (1).

3.1 "NO-GO THEOREM" (FALSE)

Let us first prove a "no-go theorem" (which will turn out to be false later) that the *naive LF restriction* of the NG-boson field leads to vanishing of both the NG-boson emission vertex and the corresponding current vertex; namely, *the NG phase is not realized in the LF quantization* [7].

Based on the LSZ reduction formula, the NG-boson emission vertex $A \to B + \pi$ may be written as

$$\begin{aligned}\langle B\pi(q)|A\rangle &= i\int d^4x e^{iqx}\langle B|\Box\pi(x)|A\rangle \\ &= i(2\pi)^4\delta(p_A^- - p_B^- - q^-)\delta^{(3)}(\vec{p}_A - \vec{p}_B - \vec{q})\langle B|j_\pi(0)|A\rangle,\end{aligned} \tag{7}$$

where $\pi(x)$ and $j_\pi(x) = \Box\pi(x) = (2\partial_+\partial_- - \partial_\perp^2)\pi(x)$ are the interpolating field and the source function of the NG boson, respectively, and $q^\mu = p_A^\mu - p_B^\mu$ are the NG-boson four-momenta and $\vec{q} \equiv (q^+, q^\perp)$. It is customary [11] to take the collinear momentum frame, $\vec{q} = 0$ and $q^- \neq 0$ (not a soft momentum), for the emission vertex of the exactly massless NG boson with $q^2 = 0$.

Then the NG-boson emission vertex should vanish on the LF due to the periodic boundary condition:

$$\begin{aligned}&(2\pi)^3\delta^{(3)}(\vec{p}_A - \vec{p}_B)\langle B|j_\pi(0)|A\rangle \\ &= \int d^2x^\perp \lim_{L\to\infty}\langle B|\Big(\int_{-L}^{L} dx^- 2\partial_-\partial_+\pi\Big)|A\rangle = 0.\end{aligned} \tag{8}$$

Another symptom of this disease is the vanishing of the current vertex (analogue of g_A in the nucleon matrix element). When the continuous symmetry is spontaneously broken, the NG theorem requires that the corresponding current J_μ contains an interpolating field of the NG boson $\pi(x)$, that is, $J_\mu = -f_\pi\partial_\mu\pi + \widehat{J}_\mu$, where f_π is the "decay constant" of the NG boson and $\widehat{J}_\mu$ denotes the non-pole term. Then the current conservation $\partial_\mu J^\mu = 0$ leads to

$$0 = \langle B|\int d^3\vec{x}\,\partial_\mu\widehat{J}^\mu(x)|A\rangle_{x^+=0}$$

$$= \quad -i(2\pi)^3\delta^{(3)}(\vec{q})\,\frac{m_A^2 - m_B^2}{2p_A^+}\langle B|\widehat{J}^+(0)|A\rangle, \tag{9}$$

where $\int d^3\vec{x} \equiv \lim_{L\to\infty}\int_{-L}^{L} dx^- d^2x^\perp$ and the integral of the NG-boson sector $\Box\pi$ has no contribution on the LF because of the periodic boundary condition as we mentioned before. Thus the current vertex $\langle B|\widehat{J}^+(0)|A\rangle$ should vanish at $q^2 = 0$ as far as $m_A^2 \neq m_B^2$.

This is actually a manifestation of the conservation of a charge $\widehat{Q} \equiv \int d^3\vec{x}\,\widehat{J}^+$ which contains only the non-pole term. Note that $\widehat{Q}$ coincides with the full LF charge $Q \equiv \int d^3\vec{x}\,J^+$, since the pole part always drops out of Q due to the integration on the LF, i.e., $Q = \widehat{Q}$. Therefore the *conservation of* $\widehat{Q}$ *inevitably follows from the conservation of* Q: $[\widehat{Q}, P^-] = [Q, P^-] = 0$, which in fact implies vanishing current vertex mentioned above. This is in sharp contrast to the charge integrated over usual space $\boldsymbol{x} = (x^1, x^2, x^3)$ in the equal-time quantization: $Q^{\mathrm{et}} = \int d^3\boldsymbol{x} J^0$ is conserved while $\widehat{Q}^{\mathrm{et}} = \int d^3\boldsymbol{x}\widehat{J}^0$ is not.

Here we should emphasize that the above conclusion is *not* an artifact of DLCQ but is inherent in the very nature of the LF quantization [8], as far as we discuss the exact symmetry limit from the beginning.

3.2 REALIZATION OF NG PHASE

Now, we propose to regularize the theory by introducing explicit-breaking mass of the NG boson m_π and then take the symmetric limit in the end under certain condition. The essence of the NG phase with a small explicit symmetry breaking can well be described by the old notion of the PCAC hypothesis: $\partial_\mu J^\mu(x) = f_\pi m_\pi^2 \pi(x)$, with $\pi(x)$ being the interpolating field of the (pseudo-) NG boson π. From the PCAC relation the current divergence of the non-pole term $\widehat{J}^\mu(x)$ reads $\partial_\mu \widehat{J}^\mu(x) = f_\pi(\Box + m_\pi^2)\pi(x) = f_\pi j_\pi(x)$. Then we obtain

$$\langle B|\int d^3\vec{x}\,\partial_\mu \widehat{J}^\mu(x)|A\rangle = f_\pi m_\pi^2 \langle B|\int d^3\vec{x}\,\pi(x)|A\rangle = \langle B|\int d^3\vec{x}\,f_\pi j_\pi(x)|A\rangle, \tag{10}$$

where the integration of the pole term $\Box\pi(x)$ is dropped out as before. The equality between the first and the third terms is a generalized Goldberger-Treiman relation, if both are non-zero. The second expression of (10) is nothing but the matrix element of the LF integration of

the π zero mode (with $P^+ = 0$) $\omega_\pi \equiv \frac{1}{2L}\int_{-L}^{L} dx^- \pi(x)$. Suppose that $\int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \pi(x)$ is regular when $m_\pi^2 \to 0$. Then this leads to the "no-go theorem" again. Thus, in order to have non-zero NG-boson emission vertex (R.H.S. of (10)) as well as non-zero current vertex (L.H.S.) at $q^2 = 0$, the π zero mode $\omega_\pi(x)$ must behave as

$$\int d^3\vec{x}\, \omega_\pi \sim \frac{1}{m_\pi^2} \quad (m_\pi^2 \to 0). \tag{11}$$

This situation may be clarified when the PCAC relation is written in the momentum space:

$$\frac{m_\pi^2 f_\pi j_\pi(q^2)}{m_\pi^2 - q^2} = \partial^\mu J_\mu(q) = \frac{q^2 f_\pi j_\pi(q^2)}{m_\pi^2 - q^2} + \partial^\mu \widehat{J}_\mu(q). \tag{12}$$

What we have done when we reached the "no-go theorem" can be summarized as follows: We first set L.H.S of (12) to zero (or equivalently, assumed implicitly the regular behavior of $\int d^3\vec{x}\, \omega_\pi(x)$) in the symmetric limit in accord with the current conservation $\partial^\mu J_\mu = 0$. Then in the LF formalism with $\vec{q} = 0$ ($q^2 = 0$), the first term (NG-boson pole term) of R.H.S. was also zero due to the periodic boundary condition or the zero-mode constraint. Thus we arrived at $\partial^\mu \widehat{J}_\mu(q) = 0$. However, this procedure is equivalent to playing a nonsense game: $1 = \lim_{m_\pi^2,\, q^2 \to 0} (\frac{m_\pi^2 - q^2}{m_\pi^2 - q^2}) = 0$ as far as $f_\pi j_\pi \neq 0$ (NG phase). Therefore the *"$m_\pi^2 = 0$ theory" with vanishing L.H.S. is ill-defined on the LF, namely, the "no-go theorem" is false*. The correct procedure should be to take the symmetric limit $m_\pi^2 \to 0$ *after* the LF restriction $\vec{q} = 0$ ($q^2 = 0$) although (12) itself yields the same result $f_\pi j_\pi = \partial^\mu \widehat{J}_\mu$, irrespectively of the order of the two limits $q^2 \to 0$ and $m_\pi^2 \to 0$. Then (11) does follow.

This implies that *at quantum level* the LF charge $Q = \widehat{Q}$ is *not conserved*, or the *current conservation does not hold* for a particular Fourier component with $\vec{q} = 0$ even in the symmetric limit:

$$\dot{Q} = \frac{1}{i}[Q, P^-] = \partial^\mu J_\mu|_{\vec{q}=0} = f_\pi \lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x}\, \omega_\pi \neq 0. \tag{13}$$

4 THE SIGMA MODEL

Let us now demonstrate [7] that (11) and (13) indeed take place *as the solution of the constrained zero modes* in the NG phase of the $O(2)$ linear σ model:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2}\mu^2(\sigma^2 + \pi^2) - \frac{\lambda}{4}(\sigma^2 + \pi^2)^2 + c\sigma, \tag{14}$$

where the last term is the explicit breaking which regularizes the NG-boson zero mode.

In the DLCQ we can clearly separate the zero modes (with $P^+ = 0$), $\pi_0 \equiv \frac{1}{2L}\int_{-L}^{L} dx^- \pi(x)$ (similarly for σ_0), from other oscillating modes (with $P^+ \neq 0$), $\varphi_\pi \equiv \pi - \pi_0$ (similarly for φ_σ). Through the Dirac quantization of the constrained system the canonical commutation relation for the oscillating modes reads [3]

$$[\varphi_i(x), \varphi_j(y)] = -\frac{i}{4}\{\epsilon(x^- - y^-) - \frac{x^- - y^-}{L}\}\delta_{ij}\delta^{(2)}(x^\perp - y^\perp), \tag{15}$$

where each index stands for π or σ. Comparing (15) with (1), we can see that the second term in $\{\}$ corresponds to subtracting the zero mode contribution out of the commutator.

On the other hand, the zero modes are not independent degrees of freedom but are implicitly determined by φ_σ and φ_π through the second class constraints, the zero-mode constraints [3]:

$$\chi_\pi \equiv \frac{1}{2L}\int_{-L}^{L} dx^- \left[(\mu^2 - \partial_\perp^2)\pi + \lambda\pi(\pi^2 + \sigma^2)\right] = 0, \tag{16}$$

and similarly, $\chi_\sigma \equiv \frac{1}{2L}\int_{-L}^{L} dx^- \{[\pi \leftrightarrow \sigma] - c\} = 0$. Thus the zero modes are solved away from the physical Fock space which is constructed upon the trivial vacuum. Note that through the equation of motion these constraints are equivalent to the characteristic of the DLCQ with periodic boundary condition [12]: $\chi_\pi = -\frac{1}{2L}\int_{-L}^{L} dx^- \, 2\partial_+\partial_- \pi = 0$, (similarly for σ) which we have used to prove the "no-go theorem" for the case of $m_\pi^2 \equiv 0$. Thus the 'no-go" theorem is a consequence of the zero-mode constraint itself in the case of $m_\pi^2 \equiv 0$. Namely, *solving the zero-mode constraint does not give rise to SSB at all in the exact symmetric case* $m_\pi^2 \equiv 0$, in contradiction to the naive expectation [6].

Actually, in the NG phase ($\mu^2 < 0$) the equation of motion of π reads $(\Box + m_\pi^2)\pi(x) = -\lambda(\pi^3 + \pi\sigma'^2 + 2v\pi\sigma') \equiv j_\pi(x)$, with $\sigma' = \sigma - v$ and $m_\pi^2 = \mu^2 + \lambda v^2 = c/v$, where $v \equiv \langle\sigma\rangle$ is the classical vacuum solution determined by $\mu^2 v + \lambda v^3 = c$. Integrating the above equation of motion over $\vec{x}$, we have

$$\int d^3\vec{x}\, j_\pi(x) - m_\pi^2 \int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \Box\pi(x) = -\int d^3\vec{x}\, \chi_\pi = 0, \tag{17}$$

where $\int d^3\vec{x}\, \omega_\pi(x) = \int d^3\vec{x}\, \pi(x)$. Were it not for the singular behavior (11) for the π zero mode ω_π, we would have concluded $(2\pi)^3\delta^{(3)}(\vec{q})\, \langle\pi|j_\pi(0)|\sigma\rangle = -\langle\pi|\int d^3\vec{x}\, \chi_\pi|\sigma\rangle = 0$ in the symmetric limit $m_\pi^2 \to 0$. Namely, the NG-boson vertex at $q^2 = 0$ would have vanished, which is exactly what we called "no-go theorem" now related to the zero-mode constraint χ_π. On the contrary, direct evaluation of the matrix element of $j_\pi = -\lambda(\pi^3 + \pi\sigma'^2 + 2v\pi\sigma')$ in the lowest order perturbation yields non-zero result even in the symmetric limit $m_\pi^2 \to 0$: $\langle\pi|j_\pi(0)|\sigma\rangle = -2\lambda v\langle\pi|\varphi_\sigma\varphi_\pi|\sigma\rangle = -2\lambda v \neq 0 \quad (\vec{q} = 0)$, which is in agreement with the usual equal-time formulation. Thus we have seen that naive use of the zero-mode constraints by setting $m_\pi^2 \equiv 0$ leads to the *internal inconsistency* in the NG phase. The "no-go theorem" is again false.

The same conclusion can be obtained more systematically by solving the zero-mode constraints in the perturbation around the classical (tree level) solution to the zero-mode constraints which is nothing but the minimum of the classical potential: $v_\pi = 0$ and $v_\sigma \equiv v$, where we have divided the zero modes π_0 (or σ_0) into classical constant piece v_π (or v_σ) and operator part ω_π (or ω_σ). The operator zero modes are solved perturbatively by substituting the expansion $\omega_i = \sum_{k=1} \lambda^k \omega_i^{(k)}$ into χ_π, χ_σ under the Weyl ordering.

The lowest order solution of the zero-mode constraints χ_π and χ_σ for ω_π takes the form:

$$(-m_\pi^2 + \partial_\perp^2)\, \omega_\pi = \frac{\lambda}{2L} \int_{-L}^{L} dx^- (\varphi_\pi^3 + \varphi_\pi\varphi_\sigma^2 + 2v\varphi_\pi\varphi_\sigma), \tag{18}$$

which in fact yields (11) as

$$\lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x}\, \omega_\pi = -\lambda \int d^3\vec{x}\, (\varphi_\pi^3 + \varphi_\pi\varphi_\sigma^2 + 2v\varphi_\pi\varphi_\sigma) \neq 0. \tag{19}$$

This actually ensures non-zero $\sigma \to \pi\pi$ vertex through (17): $\langle \pi | j_\pi(0) | \sigma \rangle = -2\lambda v$, which agrees with the previous direct evaluation as it should.

Let us next discuss the LF charge operator corresponding to the current $J_\mu = \partial_\mu \sigma \pi - \partial_\mu \pi \sigma$. The LF charge $Q = \widehat{Q} = \int d^3\vec{x}\, (\partial_- \varphi_\sigma \varphi_\pi - \partial_- \varphi_\pi \varphi_\sigma)$ *contains no zero modes* and hence no π-pole term which was dropped by the integration due to the periodic boundary condition and the ∂_-, so that Q is well defined even in the NG phase and hence annihilates the vacuum simply by the P^+ conservation [3]:

$$Q|0\rangle = 0. \tag{20}$$

This is also consistent with explicit computation of the commutators: $\langle [Q, \varphi_\sigma] \rangle = -i \langle \varphi_\pi \rangle = 0$ and $\langle [Q, \varphi_\pi] \rangle = i \langle \varphi_\sigma \rangle = 0$ [5] , which are contrasted to (6) in the continuum theory. They are also to be compared with those in the usual equal-time case where the SSB charge does not annihilate the vacuum $Q^{\rm et}|0\rangle \neq 0$: $\langle [Q^{\rm et}, \sigma] \rangle = -i\langle \pi \rangle = 0, \langle [Q^{\rm et}, \pi] \rangle = i \langle \sigma \rangle \neq 0$.

Since the PCAC relation is now an operator relation for the canonical field $\pi(x)$ with $f_\pi = v$ in this model, (19) ensures $[Q, P^-] \neq 0$ or a non-zero current vertex $\langle \pi | \widehat{J}^+ | \sigma \rangle \neq 0$ ($q^2 = 0$) in the symmetric limit. Noting that $Q = \widehat{Q}$, we conclude that the regularized zero-mode constraints indeed lead to non-conservation of the LF charge in the symmetric limit $m_\pi^2 \to 0$:

$$\dot{Q} = \frac{1}{i}[Q, P^-] = v \lim_{m_\pi^2 \to 0} m_\pi^2 \int d^3\vec{x}\, \omega_\pi \neq 0. \tag{21}$$

This can also be confirmed by direct computation of $[Q, P^-]$ through the canonical commutator and explicit use of the regularized zero-mode constraints [8].

Here we emphasize that *the NG theorem does not exist on the LF*. Instead we found the singular behavior (11) which in fact *establishes existence of the massless NG boson coupled to the current such that $Q|0\rangle = 0$ and $\dot{Q} \neq 0$*, quite analogously to the NG theorem in the equal-time quantization which proves existence of the massless NG boson coupled to the current such that $Q|0\rangle \neq 0$ and $\dot{Q} = 0$ (opposite to the LF case!). Thus the singular

[5] By explicit calculation with a careful treatment of the zero-modes contribution we can also show that $\langle [Q, \sigma] \rangle = \langle [Q, \pi] \rangle = 0$ [8].

behavior of the NG-boson zero mode (11) (or (19)) may be understood as a remnant of the Lagrangian symmetry, an analogue of the NG theorem in the equal-time quantization.

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